Exam Mesoscopic Physics 3-7-2009

Write each answer on a separate sheet. Indicate for every answer how it is obtained!
There are 6 questions.

1) Consider a ballistic constriction (point contact) in a 2 and 3 dimensional system.

   a) Derive an (approximate) expression for the (classical) conductance in 2 dimensions.
   b) Derive an (approximate) expression for the (classical) conductance in 3 dimensions.
   c) Give the expression of the conductance of a quantum point contact.
   d) Two quantum point contacts are put in parallel. They are each tuned to transmit N channels. What is the total conductance? Why?
   e) Two quantum point contacts are put in series. They are each tuned to transmit N channels What is the total conductance? Why? Does the series conductance depend on the geometry of the region in between the quantum point contacts?

2) Consider a two-dimensional electron gas with three contacts 1, 2 and 3. The contacts can each transmit N channels. A current is injected in contact 1 and taken out at contact 3. A voltage is measured between contacts 1 and 2

   a) Describe how the Landauer Buttiker formalism is applied for each of the three contacts.
   b) Write down the equations of the currents in each of the contacts in terms of reflection and transmission coefficients, the number of channels N, and the electrochemical potentials of the contacts 1, 2, and 3.
   c) Derive an expression for the measured voltage in terms of the transmission and reflection coefficients.
   d) Describe what factors determine the transmission and reflection coefficients. Assume that the two-dimensional electron gas has the shape of a rectangle with length L with contacts 1 and 2 attached to the ends, and contact 3 to the top.
   e) Give a (typical) value for the reflection and transmission coefficients for the two cases: 1) ballistic transport in the 2DEG, 2) diffusive transport with a mean free path l.

3) A one-dimensional (ballistic) wire is made in the form of a ring with radius R. The effective mass is m^*

   a) Describe how the energy levels depend on the magnetic flux. Draw the energy levels as a function of magnetic flux.
   b) Draw how the persistent current depends on the magnetic flux. Consider 3 situations where the ring contains: only one electron, two electrons, N electrons (with N a large number)
   c) The diameter of the ring is doubled. How does it change the energy levels?
   d) How does it change the persistent current?

4) A one-dimensional electron gas has a length L. The electron density is 10^{15} /m. The effective mass is m^*

a) Estimate how many electrons it contains
b) Estimate the Fermi energy
c) Estimate the average energy level spacing

5) Consider a ballistic wire made in a two-dimensional electron gas, with length L and width W. At both ends the wire is coupled to two wide regions, which act as reservoirs.

a) Make a drawing of typical electron trajectories which go through the wire, in the absence of a magnetic field.
b) Make a drawing of typical electron trajectories which go through the wire, in the presence of a perpendicular magnetic field.
c) Give an expression for the (classical) conductance of the ballistic wire in the absence of a magnetic field.
d) Make a (qualitative) drawing of the classical conductance of the wire as a function of magnetic field B. Describe how the conductance behaves for large magnetic fields.
e) The width of the wire is now doubled. Answer again question d)

6) Consider the quantum Hall effect.

a. What is the basic explanation for the quantized Hall resistance?
b. Give an argument why the quantum Hall resistance can be so accurately quantized.
   A magnetic field is used where the Landau level spacing is 1 meV.
c. Make an estimate at what temperatures the quantum Hall effect should be measured in order to be accurately quantized.